## **Plenary Lecture 1** Mathematics in the Society<sup>1</sup>

Cédric Villani<sup>2</sup>

**ABSTRACT** Mathematics is an art as old as civilization. Most of the time hidden and respected, sometimes appearing in bright light, mathematicians have always had a privileged role in society, as problem solvers, guardians of an art, deeply attached to values of intellectual freedom and opinion challenge. "The essence of mathematics lies in its freedom", said Georg Cantor. But mathematicians are also accountable to society, which is in need of keeping a link to its most singular and respected science, especially at a time of algorithmic transformation. I was lucky enough to experience the role of mathematician as a public spokesperson, advocating for mathematical sciences as both an art and a technology creator. Later, as a member of Parliament, then head of the Scientific Parliamentary Office, I experienced the intensity and complexity of science in politics, at a time when public action needs to rest on science and when human factors are more challenging than ever.

Keywords Mathematics; Science; Art; Society; Politics.

## 1. On Challenging Established Knowledge and Finding Your Way

This is Cédric Villani, speaking from Paris, delighted to be here as part of this 14<sup>th</sup> International Congress on Mathematical Education.

It's my pleasure to address to such a large audience. All my life has been devoted to sharing and transmitting knowledge, experience, advice. And in particular I'm addressing to young participants of ICME, to talk about mathematics in the society, such an important constantly renewed subject.

One day, I was invited to a very broad audience, French television show. And I knew I had just one minute to talk about the essence of mathematics. And I chose to bring with me three objects to illustrate the nature of mathematical sciences.

The first one was a book — *The Elements* of Euclid (Fig. 1a). The book sounds so familiar to mathematicians. This is the most edited book in the history of mathematics and also one of the most edited books in the whole history of the world. And also for many people like me, as a child, Euclidean mathematics, Euclidean geometry was my first contact with mathematics, with mathematics of reasoning.

The second object which I brought with me for the television show was this one, the gömböc (Fig. 1b). This Hungarian-discovered solid object, which is homogeneous,

<sup>&</sup>lt;sup>1</sup> This article is the transcript of the video record of Prof. Villany's Lecture on ICME-14, modified and approved by the lecturer.

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Fig. 1 The Elements of Euclid and the gömböc

which has only one stable equilibrium and one unstable equilibrium. So if I let it, put it whatever position you want on the table, it always come back to the stable equilibrium which is this. And there is just one unstable equilibrium which is the gömböc standing on the tip. You find all the information you want on this object. It has always fascinated the audience whenever I showed it.

First thing was the book of Euclid. The second was the gömböc. And the third one was the very familiar object which is the smartphone.

And I argued that these three objects contain the diversity, the whole narrative of mathematics. Very ancient like Euclid, and very ancient because mathematics is the only science in which discoveries are made for eternity. There is a famous description by Albert Einstein saying that this is the reason why mathematics is above all other sciences. Once you establish a truth in mathematics, it remains true forever. And the theorems of Euclid are still as true nowadays as they were before.

But also Euclid is the symbol of the time when mathematics ceased to be just rules for computation and solving problems, but also became a science of reasoning, induction.

You know the fact that mathematics wants extreme care in the reasoning. Mathematics is the only field of science in which one billion clues all going in the right direction, the same direction is not sufficient to be a proof. That's the only field of mankind's knowledge, that the only field in which you need logical reasoning, and only that to establish truths.

Now on the other hand, mathematics is not just reasoning. It's also very concrete. That's the restriction with smartphones, or with the gömböc. The gömböc is an object. The smart phone is something which allows to do a lot of things. And in one case, the gömböc is something that is just for the art. There is no application which is known of the gömböc. Nobody has made money out of it, except maybe people who make the shape and can sell it to fascinated people like me. It is beautiful but does not serve any purpose except the advancement of science and knowledge.

But on the other hand, you can make fortune on smartphone, and this is a technology which is based on mathematics.

And in math, you cannot distinguish the part which is just for the sake of the art and that part which is for the sake of applications. They go together. You cannot distinguish the concrete realization and abstract reasoning. They go together. And you cannot say that there is old mathematics and the new mathematics. They all go together. And there is a stream from the ancient past to nowadays and to the future. So you have there all the diversity. Also in the problems, the problem you can ask about Euclidean geometry, about geometry of triangles which was so fascinating for me as a student. But somebody else will be fascinated by the mysteries of the complex shapes and gömböc that appear like a challenge. Some other people will be fascinated with the applications of theory of communications and cryptography and whatever. So, the essence of mathematics lies also in the extreme diversity of motivations and applications, and of ways to do this.

And whenever in the conference you insist on the beauty of mathematics, you can be sure, that somebody in the audience will tell you: "But you know, the importance is that mathematics is useful." And whenever in the conference you insist that mathematics is useful, you can be sure that somebody will tell you that the importance is that it's also beautiful.

Mathematics is both science and technology, and both science and art. And no other science certainly, is at the same time so deeply an art. Now of course, the status of mathematicians is very much influenced by this stage of mathematics. We have to remember it as a mathematician. The mathematicians are guardians of an art. They keep the recipes, the tricks, the theories. They passed it from one generation to another. They keep alive in their brains, the discovery, the recipes. But also the mathematicians are experts in finding, discovering new ideas, new concepts, new structures, new arguments.

And this goes with the particular mentality which is always on the side of challenging opinions. New mathematics is very often obtained by challenging points of view. And it's something that I experienced in my career many times as a mathematician to make some progress. If you just follow the rules and advices given by your advisor, you are not making progress. You may obtain new computations, new applications. But you don't have new ideas. You form the mathematic art, you form the reasoning. These ones come when you challenge what comes from the lessons, coming from the master.

And through the centuries, you see so many examples of mathematicians making new discoveries by challenging the theory that was known at that time, by bringing in a new whole point of view to that. There are famous histories about this, famous stories which I often tell about, for instance, about the history of the stability of the solar system. It's one of these cases in which new theories emerged generation after generation. And the point of view changed so many times. Each time the generation of mathematicians thought with the view point that was contrary to the one of the previous generation.

Let's get started with Kepler. Kepler discovered the famous laws, named after him. He showed perfect, harmonious and simple laws, governing the solar system, an image of perfect stability. Then this was contradicted by Lagrange, Laplace and Gauss using brand new theory of arrays and perturbation theory, a masterpiece showing that stability had to occur over very large scales of time, like millions of years or something like that.

Then later it was again contradicted by Henri Poincaré. Poincaré actually thought he started to think that he could prove the long-term stability. And then he discovered, after receiving the famous prize of King Oscar of Sweden that, in fact, he could prove the possibility of instability, it was one of the famous mistakes in the history of mathematics. And after the work of Poincaré, people believe in the instability of the solar system over a very large period of time.

And then in the middle of the twenty's century, then come Kolmogorov, Arnold and Moser's beautiful work putting together classical mechanics and probability theory and Hamiltonian dynamics and new rules of probability coupled with mechanics, showing that under some assumptions (not satisfied in practice, but you could imagine applying their spirit in real life!), that had to be, with large probability, forever stability. Beautiful piece of work! And then the mood of mathematicians about the stability of the solar system had to swing again.

Then the 70's came people like Laskar and Tremaine using huge computer simulations and very clever mathematical ways of rewriting the equations showing that in practice, there should be some instability after all. So from Kepler to Laskar, you see that the mood of the dominance theory about this particular problem of solar system stability has changed many times.

Each time it comes when somebody challenges previous theory and challenges the point of view in the idea. He does not say that what the previous guys said is wrong. Remember, mathematical truths are eternal! He says "they had one point of view, but I think there is a better point of view which is this one. And this is my set of assumptions. And we try to convince you why it's the better representation of the reality and why it's better for the prediction."

This is one story. There are thousands of such stories. But this one about the stability of solar system may be the most famous because it's one of these problems in which trying to solve the problem has generated entire fields of mathematics, from matrix theory to perturbation theory, to development of probability theory, to the development of numerical computations, numerical analysis. And this is always the interplay of mathematics, the history of problems, the history of concepts. They go hand in hand.

New concepts enable to tackle new problems. New problems force you to discover new concepts. And there you go.

This story is full of adventures, full of unexpected things, full of mistakes. These are unexpected adventures. I would say, as a consequence, the mathematician always

has to be on the side of freedom of thought, freedom of speech, freedom of being able to contradict the master because this is so much engraved in our field.

Mathematician is deeply rooted in democracy, and related to democracy. And the idea is that everybody is allowed to contradict and to raise an objection.

I realized this very early, when, as a high school kid, I was confronted with mistakes of my teacher. It did happen. I remembered this course in mathematics, the teacher started to prove some theorem, etc. I saw the theorem. I could not believe it. I was maybe 14 years old, At the end of the course, I went to see the teacher and I told her "Madame, I think there is a problem with the statement that you gave us because I think I can construct a counterexample". And I showed her the counterexample. She said this was strange. I explained to her etc. She said she had to think about it. The next day and at the next course, she told me "Ok, you were right, my proof was bogus". She made a correction for the whole class. So this was an example that when you were a school kid, you are not obliged to believe in what you are told by the master. You can contradict. And if the master is of good faith, good will, the master will recognize that he or she is wrong.

In all other sciences, this is impossible. In physics, they tell you about the existence of atoms, how can you contradict the existence of atoms? You have to believe what you are told about the experiments. In biology, they will tell you about the liver, about the biology of wheat, about the sugar cycle, about the climate... whatever. You are a school kid. How can you contradict that? You have no weapon to contradict. But in mathematics, you do have the power to contradict, because everything is proven by the reasoning. You have a weapon which is your brain, your capacity of reasoning, your free spirit and ability to contradict what you are told. If you give a proof, then you can contradict the authority. I told you the example which I experienced when I was a kid.

But somehow it continued. Look at my PhD thesis. Such emotion when looking at these documents in which I put all my heart. I defended my PhD when I was 25, not so young. And it was about partial differential equations of Boltzmann, and Landau in physical theory, mathematical theory of physics of gas and plasmas. There are many chapters. My PhD started a bit like a joke. I was the president of the students' union in my university at the beginning of my PhD. I was not at all motivated by mathematics. And my advisor was worried about me. And about when I was going to get to work. Once on a small card he wrote me like "You have to, It's time for you to start working." His small document, his small card I reproduced in my PhD. I had no motivation in those days. Then I started to work and work and work. I really started to work when I was promised a good position after my PhD at the École Normale Supérieure.

Anyway, I fell in love with the Boltzmann equations which was a subject in which my PhD advisor was a very well-known master. But in my PhD, the most important parts are not the questions that were asked by my PhD advisor. These were answers to the questions that were found in the course of my PhD. In some cases, questions that my advisor liked. In some other cases, questions that my advisor did not like. Some of the techniques were radically different from the techniques of my advisor. They were

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Fig 2. Villani's book Optimal Transport, Old and New

inspired also by other people. My advisor is Pierre-Louis Lions. But I was also very much inspired by Yann Brenier, by Michel Ledoux, by Eric Carlen.

And I developed my own techniques. At the beginning of my PhD, I thought I would do the same spirit and development of what my advisor said. But by the end of the PhD, I had switched. I'd rather use different perspectives, and I had developed some other alternative techniques. And there had to be in this PhD some element of rebellion for it to be a good PhD.

In science, as in the example which I told you, a good will, a good faith, master will always recognize the merit of the student even when the student does not comply to the instructions of the master.

Also I changed subjects. I went into questions that were unorthodox. I developed some old subjects mixing it with new subjects. You know this is a great adventure when you are a young researcher, going everywhere in the world, finding new problems, being invited here, discussing with somebody, finding out there are an interesting problem, going for another collaboration etc etc etc. And this led to a number of productions, a number of new subjects.

This is my favorite, *Optimal Transport, Old and New* (Fig. 2), 1000 pages, thick book which I wrote on the subject which I did not know at the beginning my PhD. And I started to get in the subject as I started to work. By coincidence, by accident, I dipped in the subject. I was invited to some collaborations, then to give a course. Giving a course is the best way to learn your subject. That's also one possibility in which you will grow your spirit, your free spirit, and spirit of contradiction finding for yourself which are the most important things in the subject.

## 2. On the Art and Science of Mathematics

You need to always remember the sentence of Cantor, the father of the set theory, the essence of mathematics lies in its freedom. It's also freedom which guided me to maybe my most famous work, on Landau damping. The work that let me clear the bar for the Fields Medal. It is quite an accident that I started to work on this problem. Nobody asked me to, of course.

But I wanted to work on another problem regularity of Boltzmann equations. Boltzmann equation is not related to Landau damping. Landau damping is related to Vlasov equations. I wanted to continue my work on the Boltzmann equations. I had a work session with my former student Clément Mouhot — with whom I would wrote the paper eventually. I told him about my ideas. He told me his ideas. He said he remembered a conversation with Yan Guo from Providence. "Maybe there is a relation to Landau damping," he said. I never thought about that. What could it be? Then I remembered another conversation with a mathematician from Princeton. This was the start of our adventure. Then for two years, Clément and I struggled with that theorem. Not knowing at the beginning if we wanted to prove this or that, not knowing which were the tools. Making hundreds of mistakes, some of them were big, some of them were tiny, and going and going on correcting them etc. It was a great story, very fragile in the sense that many times, we thought we were facing a deadend and our proof, our reasoning could not work. In the end, we did, it did work!

Landau damping is a phenomenon of stability of plasmas very important and wellknown in plasmas physics and very important in showing applications of plasmas physics which are quite important. And it tells you that in the plasmas, you have some stability, even though the plasmas dynamics is reversible. Reversible in the sense that there are no collisions, no increasing entropy, no arrow of time. So you could think that there is no trend to equilibrium because going to equilibrium in Boltzmann equation is related to the increasing dynamics related to the arrow of time. But in Vlasov equation, there is such damping phenomenon for small perturbations. And genius physicist Lev Landau discovered this in the 30's. It was quite a shock, quite controversial.

Together with Clément, in this huge paper of ours, eventually, we showed, using new theory, new tools that indeed you could set the reasoning of Landau on rigorous mathematical footing provided that you are close enough to a stable equilibrium, provided that your perturbation is smooth enough, and it explains exactly how to compute the expected smoothness needed, making the link with a theory of Kolmogolov, Arnold and Moser, making the link with a famous experiment in plasmas physics called the echo experiment etc, and showing that different various physics phenomenon are related, even though this was not expected by physicists themselves.

So this work was full of adventures, full of wonders, full of miraculous events and so on. I was convinced in my heart that we would tackle and solve all the obstacles. But there was no logic about this belief. Some of my colleagues told me that "It was so difficult so many obstacles. How could you believe that you would solve it?" This is not something rational. It's not something that you learnt at school. It's something about passion that you have inside yourself. I said "Okay, I'm going to solve this mystery and fall in this so deeply that I will become part of the problem." In the sense that you want to solve the problem from within, in the sense that you are dreaming of the problem, with sometime some illuminations when you wake up at night etc. This is the kind of extremely intense state in which you will be.

I always say the three most important characteristics of mathematician are tenacity, imagination, and rigor. Rigor comes third. It's tenacity and imagination will make your mark and your career in the world of mathematics. This what we know when we are researchers in mathematics. But society at large is very much unaware of what it means to be a mathematician and what's the real nature of mathematics. How much passion there is in it. How much surprise there is in it. How much contradiction there can be.

It's very important at a time in which society very much relies on mathematics. It's very important for us to share about the work and the role of mathematics and mathematicians. And this is what I started to do after the Fields Medal in 2010.

I wrote a book to tell about the adventure of Landau damping problem solving. This is the book called "Living theorem" (*Théorème vivant*). The English title is *Birth of a Theorem* (Fig. 3). This is a book which is not written like mathematics book, because there are portraits of mathematicians and there are stories like autobiography a little bit. During a short period of time, during the two years of working on the problem, there are email exchanges, some of them very tormented you know... It's not like usual book explaining about science for broad audience because there are also formulas. There is a portrait of Carlo Cercignani and I explain what was his conjecture. What were the tools that we setup to prove it etc etc. And also I put in the book some of the big formulas that we have to fight with as mathematicians.



Fig. 3 *Théorème vivant* and its English and Chinese versions

And this was, you know, to show in the impressionist way what is the life of the mathematician. Also about cultural aspects like what is it that you do at night, how you

work, what kind of the songs you listen to, etc. There is a part of book in which I describe the songs. There is a part of the book in which I describe encounter with various mathematicians etc etc. See, browsing through the pages of the book... This is about the songs... This is about some of the formulas... You know, to show in an impressionist way the life and work of mathematicians, insisting that mathematics is not a deterministic science but also a passionate art.

It's as if your kid wants to know about your work as mathematician. You don't try to simplify things. You bring your kid to work. And the kid will see what you are doing, with whom you are talking. How you are desperate sometimes, happy sometimes. What you are listening to etc. The whole life of a mathematician.

The book, precisely because of the original point of view, had quite a success. It was translated in many languages, including Chinese. An important thing to know about the book is that I was not my idea in the first place. It came by accident by encountering a famous editor in a dinner, several months before the Fields Medal. The editor was attracted by my spider. We talked to engage conversation, what you do, etc. "I was told that you are a famous mathematician."

He wanted to have me write a book for him. But all the examples of books that I could think of were of no interest for him. "I can write you a book on Boltzmann equation", "I don't care", "A book on the Entropy", "I don't care," "I can write you a book on information etc", "No". And he told me "I want to know how you work, how you think as a mathematician. What goes on in your head". I was very much embarrassed. I thought about it. Then the result was the book.

I thought I'm going to write it. It will be what is behind the curtains when you make a discovery of a theorem. And what is the process of being a mathematician. And I will tell it like an adventure. Actually, the editor thought he could even publish it as a novel.

This was the start of many years of communications of mathematics which I undertook in collaboration with various people with always the same idea that to make people understand mathematics, love mathematics. Don't just talk to them about how useful is mathematics. Tell them about the adventure of being a mathematician, about the art.

Tell it like it's a bunch of stories. And stories always mixing three traits, the stories of people, the stories of problems, the stories of concepts. And when you make a lecture, a good lecture, it has to be these three traits together. Maybe the idea of Fourier will come here, in the problem and another problem. Problem that Fourier wanted to solve. And the life of Fourier himself. How he was also in the revolution or after the revolution, the empire etc. And all this makes mathematics an art, a human activity as well as sciences.

I did this for so many projects. For instance, here is a comic strip (Fig. 4a) which I wrote with the great French comics drawer Edmond Baudoin, very famous. It talks about some very important hidden characters involved in science or innovation during



Fig. 4 Some "non-mathematics" cooperative projects

the Second World War who had a lot of influences through their discoveries, their mistakes, their actions. The kind of spirit they were in, the kind of crazy story that they had to live. And what is in their heads when they are afraid, when they are proud, when they are shamed, when they are full of questions, and so on. So these were Heisenberg, and Turing, and Szilard and Dowding. And telling this as an adventure as well as science in the construction.

Then there's a little bit more crazy project (Fig. 4b) made together with the great photographer Lisa Roze, very particular very special photographs. And she wanted me to write the text corresponding to the photographs. And she was attracted by collaboration with me because she thought highly of my imagination. And she thought I was ready to find some logic behind her photographs, to find some way to order them, to find some text with some kind of boutique twists etc.

This is not mathematics of course. This is contributing to the idea that mathematics comes with imagination as an art.

It's also something which I insisted on the series of standup lectures in the Maison des Métallos, a Culture hall in Paris, very much about bringing quality culture to all kinds of audience, including audience which do not usually get interested in mathematics. Look at the illustrations: You see the gömböc... this is an orrery displaying the motion of planets... the geology... this is a twin surface with negative curvature, whatever.

So in all these lectures, always, it was about the art of it, telling stories, and always stories about bringing together these three straits, story of ideas, story of people, story of concepts. My book being dedicated to a very inspiring mathematical figure that was Maryam Mirzakhani.

So this for a number of years, was the main of my activities, talking about mathematics, making collaborations of mathematicians with artists, finding ways to

talk about problems, but also get involved in some projects, some associations, and so on, and being part of society. I strongly believe that we as mathematicians, have to get very much inside the society. This is also something that was one of the main characteristics of the career of my PhD advisor, Pierre-Louis Lions.

## 3. From Science to Society to Political Decision

Learning to do mathematical communications was not immediate. I had to work hard to find my ways to do mathematical outreach, speak about mathematics to large audience.

It was very important that for many years, I was a professor in Lyon. My neighbor was Étienne Ghys, one of the first masters of mathematical communication. At some point, he was considered the best mathematics lecturer in the world. His lectures in ICM and so on were extraordinary. And he took the job of mathematical outreach extremely seriously and I was very much influenced by that.

In Lyon, there was an atmosphere, both at the École Normale Supérieure in Lyon and in the Lyon I University, there was the atmosphere about sharing to the society, giving account to the society which is legitimate because we are part of the society, we are often funded by the society by public money. But also we need to get from the society the problem, the people, whatever. We are part of the society, we need to find a good way to underline it.

My first interview for a journalist's journal (not a science journal!) was a disaster. The journalist wanted to cancel the interview because it was so obscure, whatever. And then I worked. I went to, you know, learning sessions, master classes whatever and was passionate about that also. I wrote in a newspaper some columns. I was in the radio. I learnt a lot. And there was all these collaborations and the projects that I told you about.

Some of the most important lessons I learnt is that first to attract your audience you need to talk to the heart first, emotions. Something which looks close to you, which will bring a link between the audience and you. Something related to the culture. Culture is what brings people together. Emotion is what brings people together.

Then after you hooked your audience, with these emotions, you can go on with the concepts, with the stories, with the rationality, with the logics, you have to start with the irrational thing to speak. Never separate the concepts from human, human contact, human adventure and so on.

This was somehow in the direction, the continuation of my mathematical work. Then I did more than that and went further on. Becoming a politician and going into politics for real in 2017. Okay actually 2017 was not my first encounter with politics. As soon as 2010, right after the Fields Medal, I started to be engaged in a number of projects, associations, institutes, government, talking with people with all kind of responsibility.

I always had some faithful responsibility. This certainly goes back to the time when I was 21 years old, and I had myself elected as the head of students' office. And I likes it so much, handling the problems, finding how to try to create the cohesion. After 2010, I was involved in political groups about the construction of Europe, federalist as they are called. I still consider myself as a federalist, name given to those people who believe all nations in Europe have to engage in the strong integration process so that Europe can become a strong unity. A political entity, an economical entity with some cohesion and some political strength. This was back in 2010. But this led me little by little to be more involved with the national politics.

In 2017, with some, you know, some complicated sequence of events, I ran for Parliament. And I was elected. It was a new life that started, new life in which the big problem was how to make sense of my scientific career within the political career. Convinced that each makes sense, to have science and politics, but it's a delicate equation to find the problems and ways in which your scientific experience can be useful. International experiences that goes within research, with the problem solving. That is quite important in politics. Also the fact that, politics nowadays raises a lot of very tricky scientific problems, related to environment, related to pandemics, related to biology, related to energy whatever.

And in these four years, from 2017 to now, on some occasions, I managed to make the better of my scientific experiences. In some other occasions, I did not manage. Whether it was a success or a failure, it was always instructive. Some of the most important things which I think I let to do is the mission on artificial intelligence. I'm showing it to you. This is the journal in which this paper in which there was the interview like "live my life of member of parliament". And it was in those days, when there were many newcomers in politics in 2017, the interviews like "How was it to be in parliament, what do you think". At first in parliament, other members of parliament would look at me as a strange beast, you know. What does this mathematician do, I'm on gas.

There was a time in history in which mathematicians were quite numerous, scientists were quite numerous in the French political life. But this was long ago. There were many examples long ago. Nowadays very few examples.

And one of the first missions that was given to me, for six months, was a mission about artificial intelligence. This is a report which I wrote after six months of work with my team. A report which tries to get a complete picture about the artificial intelligence with consequences for France and for Europe, about data policy, about the ethics of AI, about the job future, about education, about fields of applications that should have priority like health and mobility, environment and defence...

And you know, trying to make the big picture, the whole picture of what AI should be like and why it's the business of everybody. So this report was for writing a technical document, but also for addressing broad audience, going in the TV to talk about AI, going to newspapers to talk about AI, going to debates to talk about AI. Of course, it rested on my experiences of sciences as mathematician.

For instance, this is a book (Fig. 4) which I edited together with medical doctor Professor Bernard Nordlinger, about health and artificial intelligence. I'm writing this on behalf of the academy of sciences, Nordlinger on behalf of the academy of medicine. And we organize conferences, we invite lectures, whatever. So this is the scientific part.

Based on the scientific part, I wrote this report. Nowadays, I still get asked sometimes by members of the government, sometimes by people from the industries, sometimes by just citizens about AI. Asking what I think about this etc. Recently, there was a delegation from people from America to talk and discuss about American report and compare it to the French report and etc. So this is an example in which the scientific knowledge could be brought directly to the heart of the policy and society problem.

The plan which was deduced from this book, from this report has been implemented too, in large part, still a number remains to be done. It became the official plan of AI for the French government.



Fig. 4 A book on health and AI edited by Bernard Nordlinger, Cédric Villani et al.

Another example which was more tricky, was a report on the teaching of mathematics in France (Fig. 5). A work I did with math teacher and inspector Charles



Fig. 5 The report on the teaching of mathematics in France by Villani and Torossian (https://www.vie-publique.fr/sites/default/files/rapport/pdf/184000086.pdf)

Torossian. To show and see what we had to do to repair the French system of mathematics teaching, where were the main difficulties? How to act? Training of teachers on the curriculum, on the tools, on the ways to organize things in high school, in medium high school, in elementary school, etc.

The report went very well. The report was very much praised by the mathematician community, and by the ministry of education and was started to be implemented. And then problems began. The ministry of education did not implement just our report but also a number of other things which had some constraints which were determined by this or that etc. An enormous number of difficulties arose in which I had no role and no part. I did write some warnings for the government and said "okay be careful about this, this and that", but in the end, the situation is extremely confused right now in particular among mathematicians and mathematics teachers.

It's good to write a report. But you always have to remember implementation is much more difficult. I did regret that I was not more involved in the implementation in this case. Maybe some misunderstanding could have been avoided. And currently there's a big number of things to fix still in the mathematics strategy.

Another of the things which I did and which I think was badly needed was the reform of the scientific parliamentary office. More precisely, parliamentary office of evaluation of scientific and technological choices: it is a group made of 18 members of National Assembly and 18 members of Senate. So both chambers of the Parliament working on problems which are of scientific or technological nature, important for enlightening the choice of the politics on it. Say, politicians have to make choices of nuclear energy, but this is a highly technical subject in which you need the opinions of many scientists. We need to organize some contradictory debates and also to hear the point of view from the society etc. That's the role of scientific parliamentary office, organize these debates on problems which are the interfaces of science and politics.

And it was funded in the 80's. It was just funded precisely to work on the subjects related to nuclear energy. So I became the president of this office. And I worked a lot to diversify the themes of the office, to hire scientists within this parliamentary office, to go more in the directions of humans in social sciences, to get more in association with the working groups of the Parliament, to follow more closely the base of the political event, etc. Quite a work.

What subject did we treat recently? Things such as research in polar environment, long-term Covid, or scientific integrity, or viruses used to fight bacterial diseases, or quantum computing and quantum information and quantum cryptography, or open science, or all kinds of therapeutic subjects, you name it. One of the trickiest subjects which we had to handle is the about the new breeding technologies. These new techniques of biotechnology based on editing genome techniques such as CRISPR-Cas9 to edit the genomes of plants. So there are aspects related to biology, related to economics, related to ethics. Lots and lots of debates. In this case for NBT, it was a very hard debate. The two parliament members who were responsible of the subject

had conflicting opinions, could not get together. And I, as the president of the office had to step in and organize syntheses with the help of the scientific secretary. We were discussing for a long time on each subject. And we managed to formulate seven propositions, seven recommendations. The two MPs in charge agreed on six of the seven recommendations. And on just one, the seventh one, they had conflicting opinions. And we could exactly pinpoint why and what kind of different values it's corresponding to: Then we have made our job for the benefit of the Parliament. Not taken a decision because it's for the democratic decision to proceed, through the democratic procedure un Parliament. But we studied the subject and analyzed it to the point where a political decision becomes possible by analysing the possible choices. Balancing the various problems and advantages or drawbacks, balancing the natural sciences with the human sciences etc.

A common feature with all these problems that we treated, in fact, is that the most tricky parts are those related to social and human sciences, like opinion issues, bubbles of information, fake news. In science, we are used to the idea that no one a priori has the truth, we have to listen to all the contradictions. Even revered scientists can make mistakes. And there are famous examples in the history of science in which the best scientist of the world makes horrible mistakes, and in fact, some of his opponents was right. We know such examples. Eventually, all these problems get resolved in the right direction. But sometimes it takes time. But one should never a priori believe as a god given truth what somebody or some entity or some institution or some political party says. One always has to think "is this really true? Can I contradict? Where is the logical reasoning?" etc etc. That's what we do at the scientific parliamentary office organizing the political debate, which is the basis of democracy. The heart of democracy is the contradictory, open, sincere debate of ideas.

And I tell you and this would be my conclusion. In a world in which debate of ideas are extremely confused, in a world in which many political problems hugely rely on scientific and technological advances and choices, in a world in which science is hugely used by the power, we, scientists have huge responsibility to get committed in the world and to make sure, but we keep the memory of the past and everything that we can learn from the past, from the history of science, and from history of mankind. And that we get committed to making our vision of the future explicit, saying which are the consequences of the choices that we will make, consequences on our children. What we should transmit to the next generation. Currently, more than ever, and in a world which is more mathematical than ever with the advance of information, technologies, of statistics everywhere, of artificial Intelligence, you name it, all kinds of mathematical application in the world, we, mathematicians have a responsibility of being committed to the society for the future and for the sake of the future generations.

Thank you for listening.